DISTRIBUTION OF SUSPENDED PARTICLES IN A TURBULENT FLOW

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A steady-state turbulent flow in a plane slit or in a boundary layer serve as examples by means of which we examine the profiles of concentration and velocity of fine suspended particles.

The problem of particle distribution in agitated suspensions and in suspensions in gas has been covered extensively in the literature. Although the physical mechanisms responsible for the formation of the concentration and velocity profiles of the dispersed phase are known, on the whole, at the present time we have no generalizing and sufficiently noncontradictory models. At the same time, these profiles exhibit a number of extremely nontrivial features that are significantly dependent on the parameters of the flow and the physical characteristics of the phases, and where the dispersion-phase content is not overly small they are capable of seriously affecting the hydrodynamics and transport processes within the mixture. The characteristic concentration profiles in steady-state one-dimensional flows have been described, for example, in [1-7]; depending on the properties of the mixture phases and those of the flow, these may correspond to an accumulation of particles both in the central and peripheral regions of the flow. Analogous information with respect to the profiles of particle velocity can be found in [6-9]; we normally note a lagging of the particles from the fluid in the core of the flow, whereas they precede the fluid in the region near the wall.

For the sake of simplicity, in the following we will examine only the steady-state flow in a plane channel or in the boundary layer on a plane plate, neglecting the force of gravity. Moreover, we will assume the particles to be fine, and we will further assume that their weight and volume concentrations are small. First of all, this allows us to assume approximately that the particles are totally attracted to the turbulent moles and, secondly, it allows us to neglect the reverse influence of the dispersion phase on the characteristics of turbulence, assuming the latter to coincide with the characteristics for the corresponding single-phase flow.

The Reynolds equations, together with the assumptions adopted above, can be written in the form

$$0=-\frac{\partial p}{\partial x}+\frac{d}{dy}\left(\mu_0\frac{dv}{dy}-\rho_0\langle v_x' v_y'\rangle\right)-f_x,$$
$$0=-\frac{\partial p}{\partial y}+\frac{d}{dy}\left(-\rho_0\langle v_y'^2\rangle\right)-f_y \tag{1}$$

for the continuous phase and in the form

$$0=\frac{d}{dy}\left(-\rho_1\varphi\langle v_x' v_y'\rangle\right)+f_x,\quad 0=\frac{d}{dy}\left(-\rho_1\varphi\langle v_y'^2\rangle\right)+f_y \tag{2}$$

for the dispersion phase; the longitudinal component f_x and the lateral component f_y of the interphase interaction forces have been referred to a unit volume of the mixture. Using the Stokes formula, we have

$$f_x=\beta\varphi(v-w),\quad \beta=9\mu_0/2a^2. \tag{3}$$

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We will estimate the lateral component of the force on the basis of the familiar Saffman formula [10, 11] for the force acting on the particle in shear flow. This gives us

$$f_y = \gamma\varphi \left|\frac{dv}{dy}\right|^{1/2} (v - w),\ \gamma = \frac{4.8}{\pi} \frac{\rho_0' \sqrt{\nu_0}}{a}: \tag{4}$$

the limitations imposed on the applicability of this formula are discussed below.

Bearing in mind that in accordance with the assumption that $\rho_1\varphi \ll \rho_0$, we assume $\partial p/dx = \tau/L = \rho_0 u_*^2/L$. Combining the first of the equations in (1) and (2), and taking into consideration that when y = 0 (i.e., at the wall) $\langle v_x' v_y' \rangle = 0$, after integration we have

$$\langle v_x' v_y' \rangle = \frac{u_*^2}{L} y + \nu_0 \left(\frac{dv}{dy} - \frac{dv}{dy}\Big|_{y=0}\right). \tag{5}$$

It follows further from the first of the equations in (2) that

$$v - w = \frac{\rho_1}{\beta\varphi} \frac{d}{dy} \left\{ \varphi \left[\frac{u_*^2}{L} y + \nu_0 \left(\frac{dv}{dy} - \frac{dv}{dy}\Big|_{y=0} \right) \right] \right\}. \tag{6}$$

Thus, the distribution of the tangential Reynolds stress and the velocity of interphase slippage through the cross section of the flow are uniquely defined in terms of the velocity profiles of the corresponding single-phase flow for which we must make use either of the theoretical or an empirical representation. For the sake of definiteness, we will examine the Van Driest model below [12, 13], which exhibits a certain theoretical foundation [14], i.e., we will assume that

$$v = u_* \int_0^\eta F(\eta)\, d\eta,\quad \eta = \frac{u_* y}{\nu_0},\quad F(\eta) = \frac{2}{1 + f(\eta)},$$
$$f(\eta) = \{1 + 4\varkappa^2 \eta^2 [1 - \exp(-\eta/A)]^2\}^{1/2}, \tag{7}$$

where $\varkappa \approx 0.4$ [12] and $A \approx 30$ [14] are numerical coefficients. With consideration of (7), from (5) and (6) we have

$$\frac{\langle v_x' v_y' \rangle}{v_*^2} = \frac{y}{L} + \frac{1 - f(\eta)}{1 + f(\eta)},\quad \eta = \mathrm{Re}_* \frac{y}{L},\quad \mathrm{Re}_* = \frac{u_* L}{\nu_0},$$
$$\frac{v - w}{u_*} = \frac{\rho_1 u_*}{\beta\varphi} \frac{d}{dy} \left\{ \varphi \left[\frac{y}{L} + \frac{1 - f(\eta)}{1 + f(\eta)} \right] \right\}. \tag{8}$$

The corresponding representations for the flow in the pressure-free turbulent boundary layer are formally obtained out of (8) as $L \to \infty$. Let us note that for purposes of simplification no provision is made in (7) and (8) for the absence of velocity in the central region of the slit nor for the intermittence of turbulence at the outside edge of the boundary layer.

With both large and small η the following asymptotic formulas follow from (8):

$$\frac{\langle v_x' v_y' \rangle}{u_*^2} \approx -\left(1 - \xi - \frac{1}{\varkappa \mathrm{Re}_* \xi}\right),\ \xi = \frac{y}{L} \gg \frac{1}{\mathrm{Re}_*},$$
$$\frac{\langle v_x' v_y' \rangle}{u_*^2} \approx \xi \left(1 - \frac{2\varkappa^2}{A^2} \mathrm{Re}_*^4 \xi^3\right),\ \xi = \frac{y}{L} \ll \frac{1}{\mathrm{Re}_*}, \tag{9}$$

the first of which corresponds to the logarithmic velocity profile and corresponds to the observed behavior of the tangential Reynolds stress outside of the viscous and transition layers [12, 15]. The second asymptote shows that on approach to the wall this stress tends to vanish, but by no means monotonically, and there exists a region in which it is positive.

In order to gain some judgment as to the nature of the interphase slippage in the flow, from (8) let us calculate the relative volume flow of the dispersion phase in a wall layer of thickness y. Using the definition for β in (3), we obtain

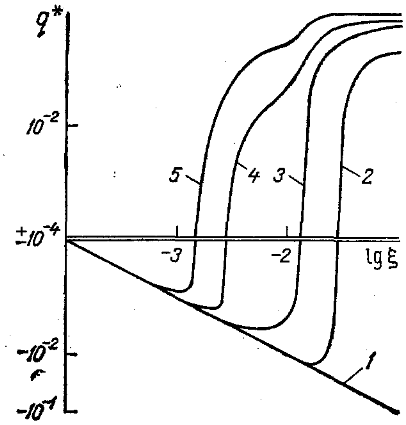

Fig. 1. Dimensionless relative flow $q^* = q/q_0Q\varphi$ as a function of the dimensionless coordinate $\xi = y/L$: 1-5) $Re_* = 10, 100, 250, 500, 1000$.

$$q(y) = -\int_0^y \varphi(v-w)\,dy = -q_0 Q\varphi\left[\xi + \frac{1-f(\xi\,\mathrm{Re}_*)}{1+f(\xi\,\mathrm{Re}_*)}\right], \tag{10}$$

$$q_0 = au_*,\ Q = (2/9)(\rho_1/\rho_0)(a/L)\,\mathrm{Re}_*.$$

This formula is shown in Fig. 1. If $d\ln\varphi/dy \ll 1$, the extrema of q(y) correspond to the zeros of velocity $v - w$, which in this case can be represented in the form

$$\frac{v-w}{u_*} \approx -\frac{1}{\varphi}\frac{dq}{dy} = U\,\mathrm{Re}_*\left\{1 + \mathrm{Re}_*\frac{d}{d\eta}\left[\frac{1-f(\eta)}{1+f(\eta)}\right]_{\eta=\xi\mathrm{Re}_*}\right\}, \tag{11}$$

$$U = (2/9)(\rho_1/\rho_0)(a/L)^2.$$

For large and small $\eta = \xi\,\mathrm{Re}_*$ from (10) and (11) we obtain formulas analogous to (9). For example,

$$\frac{v-w}{u_*} \approx U\left(1 - \frac{1}{\varkappa\,\mathrm{Re}_*\,\xi^2}\right)\mathrm{Re}_*,\ \xi \gg \frac{1}{\mathrm{Re}_*},$$
$$\frac{v-w}{u_*} \approx U\left(1 - \frac{8\varkappa^2}{A^2}\mathrm{Re}_*^4\xi^3\right)\mathrm{Re}_*,\ \xi \ll \frac{1}{\mathrm{Re}_*}. \tag{12}$$

We see that the fluid precedes the particle in the central region $y_2 < y < L$, in the case of $\mathrm{Re}_* \gg 1$ occupying virtually the entire cross section of the flow, and also in the immediate vicinity of the wall, when $0 < y < y_1$. In the intermediate layer $y_1 < y < y_2$ the particles move ahead of the fluid. The simplest estimate of the boundary for these regions can be obtained from (12):

$$\xi_1 = \frac{y_1}{L} \approx \left(\frac{A^2}{8\varkappa^2}\right)^{1/3}\frac{1}{\mathrm{Re}_*^{4/3}},\ \xi_2 = \frac{y_2}{L} \approx \frac{1}{(\varkappa\,\mathrm{Re}_*)^{1/2}}. \tag{13}$$

Immediately at the wall the velocity v is as small as need be, because of the condition of adhesion, while w is negative and in order of magnitude equal to $u_*U\,\mathrm{Re}_*$. Hence it becomes clear that the particles at the wall (on the average, of course) move in a direction opposite to that of the flow, without any relationship to the velocity of the latter. This reverse motion of the particles at the walls has been experimentally observed [16, 17]. (Let us stress that we have not taken into consideration here the contact interrelationships between the particles and the solid wall, these being capable of considerably affecting the result under consideration here. Moreover, our conclusion loses any significance for large Re_* numbers, when y_1 becomes comparable to a.)

In the earlier approximation $d\ln\varphi/dy \ll 1$ from (7) and (11) we also derive a formula for the ratio of the particle and fluid velocities, averaged over the cross section of the slit,

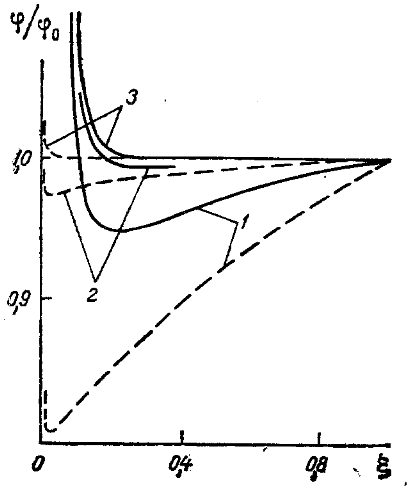

Fig. 2. Distribution of particle volume concentration through the cross section of the slit for the case in which $\log \mathrm{Re}_* = 3$ and 4 (the solid and dashed curves, respectively); 1-3) $\log \Pi = 3, 4, 5$.

$$\langle w\rangle/\langle v\rangle = 1-\delta,\ \delta = (\langle v\rangle - \langle w\rangle)/\langle v\rangle \approx U\,\mathrm{Re}_*\,F(\mathrm{Re}_*)\left[\frac{1}{\mathrm{Re}_*}\int_0^{\mathrm{Re}_*}\left(\int_0^{\eta} F(\eta)\,d\eta\right)d\eta\right]^{-1}. \tag{14}$$

The velocity of the interphase slippage as a function of the physical and regime parameters qualitatively agrees with the experimental data (obtained primarily for turbulent flows in circular tubes [6-9]). The slippage effect is intensified as the dimensionless ratio ρ_1/ρ_0 and a/L and the dynamic Reynolds number are increased, i.e., the velocity of the flow. However, within an increase in these two ratios the assumption of complete attraction of the particles by the turbulent moles becomes unacceptable. Moreover, the reduction in the pressure of the gas in the gas suspension leads to a breakdown in the assumption to the effect that the weight concentration of the dispersion phase is small: in this case, in the place of ρ_0 it is more correct to use the quantity $\rho_0 + \rho_1\langle\varphi\rangle$ as the effective densities. On the whole, this leads to considerable weakening of the influence exerted by these parameters on the slippage effect. Let us also note that formulas (11) and (14) become excessively course with an increase in Re_* and a/L, because of the formation of a significantly nonuniform concentration profile with a tendency of particle accumulation in the center of the slit and in the region in which the particles precede the fluid (see below).

Let us now examine the particle concentration profiles. From the second of the equations in (1) and (2), with consideration of (4), (7), and (8), utilizing the above-stipulated simplifications, we obtain:

$$\frac{\gamma}{\beta}\left[\frac{u_*^2}{v_0}F(\xi\mathrm{Re}_*)\right]^{1/2}\frac{d}{dy}\left\{\varphi\left[\xi+\frac{1-f(\xi\mathrm{Re}_*)}{1+f(\xi\mathrm{Re}_*)}\right]\right\} = \frac{1}{u_*^2}\frac{d}{dy}(\varphi\langle v_y'^2\rangle),\quad \frac{\partial}{\partial y}(p+\rho_0\langle v_y'^2\rangle)=0. \tag{15}$$

From this it follows, in particular, that $p + \rho_0\langle v_y'^2\rangle = \mathrm{const}$, i.e., for the determination of $\langle v_y'^2\rangle$ it is necessary to know the distribution of the static pressure p through the cross section of the flow. In the region of developed turbulence, i.e., outside of the viscous and transition layers, the relationship between p and y can, apparently, be neglected. We then have $\langle v_y'^2\rangle = bu_*^2$, where b = const. From the Cont-Bello experimental data [15], the coefficient b diminishes in reality slowly from ~1 when $\xi \leq 0.4$ to ~0.7 when $\xi \approx 1$. Here, bearing in mind the derivation of the approximate results, we will neglect the change in b in this region, and $\langle v_y'^2\rangle$ as a function of y within the limits of the viscous and transition layers is determined on the basis of the results from [14], which yield

$$\langle v_y'^2\rangle = b[1-\exp(-\xi\mathrm{Re}_*/A)]^2 u_*^2. \tag{16}$$

From (15), with consideration of the definitions for β and γ in (3), (4), and (16), we have

$$\frac{d\ln\varphi}{d\xi} = \left\{1-\frac{d}{d\xi}\left[\frac{f(\xi\mathrm{Re}_*)-1}{f(\xi\mathrm{Re}_*)+1}\right]-\frac{2\Pi}{A}\left[\frac{f(\xi\mathrm{Re}_*)+1}{2}\right]^{1/2}\times\right.$$

$$\times\left[1-\exp\left(-\frac{\xi \mathrm{Re}_*}{A}\right)\right]\exp\left(-\frac{\xi \mathrm{Re}_*}{A}\right)\right\}\left\{\frac{f(\xi \mathrm{Re}_*)-1}{f(\xi \mathrm{Re}_*)+1}-\xi+ \tag{17}$$

$$+\frac{\Pi}{\mathrm{Re}_*}\left[\frac{f(\xi \mathrm{Re}_*)+1}{2}\right]^{1/2}\left[1-\exp\left(-\frac{\xi \mathrm{Re}_*}{A}\right)\right]^2\right\}^{-1},$$

$$\Pi=(3\pi b/3{,}2)(L/a).$$

If we take $\eta \approx 30$ [12] as the dimensionless coordinate of the external boundary of the transition layer, then in the region $30/\mathrm{Re}_* \approx \xi_0 < \xi < 1$ in the place of (17) we can use the simple asymptote

$$\frac{d\ln\varphi}{d\xi}\approx\left(1-\frac{1}{\varkappa \mathrm{Re}_*\xi^2}\right)\left[1-\xi-\frac{1}{\varkappa \mathrm{Re}_*\xi}+\frac{\Pi\sqrt{\varkappa\xi}}{\sqrt{\mathrm{Re}_*}}\left(1+\frac{1}{4\varkappa \mathrm{Re}_*\xi}\right)\right]^{-1}. \tag{18}$$

The value of $\xi = \xi_2 = (\kappa \mathrm{Re}_*)^{-1/2}$, corresponding to the change in sign of the numerator in the right-hand side of (18), in the case of $\mathrm{Re}_* \gg 1$, obviously falls within the region for which this asymptote is valid. The denominator of this quantity changes sign only with sufficiently small Π at the point $\xi \le \xi_3$, where ξ_3 denotes its root when $\Pi = 0$. If $\mathrm{Re}_* \gg 1$, $\xi_3 \approx (\kappa \mathrm{Re}_*)^{-1} < \xi_0$. Thus, in the ξ interval under consideration the denominator in the right-hand side of (18) is positive. From (18) we obtain the following for the core of the flow:

$$\varphi\sim\exp\left[-(2/\Pi)\sqrt{\mathrm{Re}_*/\varkappa}\,(1-\sqrt{\xi})\right],$$

i.e., in the center of the slit we have a maximum for φ, all the more pronounced, the smaller Π and the larger Re_*. The smoothness condition when $\xi = 1$ is not satisfied because no provision has been made for the absence of a flow velocity. In the region in which $\xi \sim \mathrm{Re}_*^{-1/2}$, when $\Pi \ll \mathrm{Re}_*^{3/4}$, we have

$$\varphi\sim\exp(\xi+1/\varkappa \mathrm{Re}_*\xi),$$

while when $\Pi \gg \mathrm{Re}_*^{3/4}$

$$\varphi\sim\exp\left(\frac{2}{\Pi}\sqrt{\frac{\mathrm{Re}_*\xi}{\varkappa}}+\frac{2\mathrm{Re}_*}{3\Pi}\frac{1}{(\varkappa \mathrm{Re}_*\xi)^{3/2}}\right);$$

in either case φ exhibits a minimum when $\xi = \xi_2 = (\kappa \mathrm{Re}_*)^{-1/2}$. With a further reduction in ξ, i.e., on approach to the wall, the particle concentration increases with extreme rapidity.

The concentration profiles obtained by numerical integration of Eqs. (17) and (18) are shown in Fig. 2. We see that in the region at the wall a concentration maximum is achieved. From a qualitative standpoint, these profiles are not in poor agreement with the newest experimental data, describing virtually all of the observed types of distributions [1-7]. Thus the saucer- and saddle-shaped profiles correspond to large Π and small Re_*. With an increase in Re_* when $\Pi \gg 1$ the region of virtually uniform particle distribution expands, encompassing almost the entire cross section of the slit. As Π is reduced, we find a transition from the uniform to the trapezoidal and to the cupola-shaped. (Here we employ the terminology of [7] and the Herman dissertation, used in the cited reference.)

Equations (17) and (18) allow us to reject the hypothesis to the effect of virtually uniform particle distribution in the flow, such as was employed in the derivation of formulas (11)-(14). Within the entire flow region we have

$$\frac{v-w}{u_*}=U\mathrm{Re}_*\left\{1-\frac{d}{d\xi}\left[\frac{f(\xi \mathrm{Re}_*)-1}{f(\xi \mathrm{Re}_*)+1}\right]-\left[\frac{f(\xi \mathrm{Re}_*)-1}{f(\xi \mathrm{Re}_*)+1}-\xi\right]\frac{d\ln\varphi}{d\xi}\right\}, \tag{19}$$

and $d\ln\varphi/d\xi$ is determined from (17). In the region of developed turbulence from (19) with consideration of (18) we have

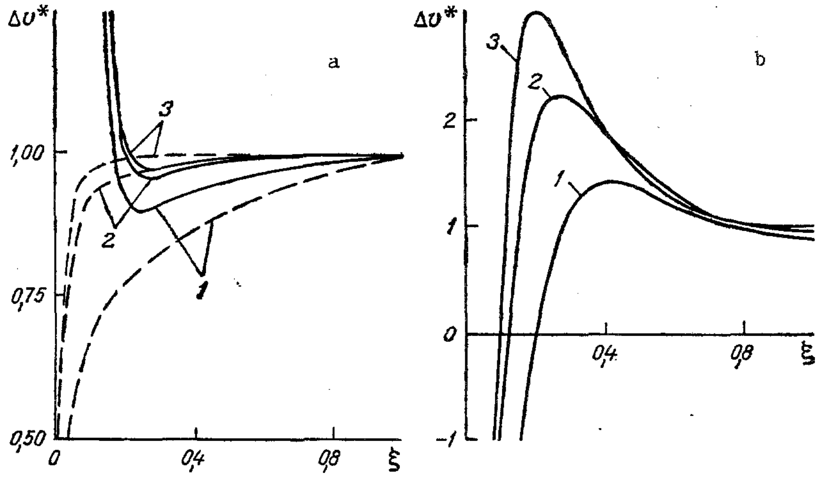

Fig. 3. Profiles of dimensionless rate of phase slippage $\Delta v^* = (v - w)/u_*URe_*$ in the cross section of the slit: a) $\log Re_* = 3$ and 4 (solid and dashed curves, respectively); 1-3) $\log \Pi = 3$, 4, 5; b) 1-3) $Re_* = 50$, 75, 100, $\log \Pi = 4$.

$$\frac{v-w}{u_*} \approx U\,\mathrm{Re}_*\left(1-\frac{1}{\varkappa\,\mathrm{Re}_*\,\xi^2}\right)\left(\frac{\varkappa}{\mathrm{Re}_*}\right)^{1/2}\Pi\sqrt{\xi}\left(1+\frac{1}{4\varkappa\,\mathrm{Re}_*\,\xi}\right)\times$$
$$\times\left\{1-\xi-\frac{1}{\varkappa\,\mathrm{Re}_*\,\xi}+\left(\frac{\varkappa}{\mathrm{Re}_*}\right)^{1/2}\Pi\sqrt{\xi}\left(1+\frac{1}{4\varkappa\,\mathrm{Re}_*\,\xi}\right)\right\}^{-1}, \tag{20}$$

and here the corresponding formula from (12) obviously corresponds to the limit $\Pi \to \infty$. The distributions of the dimensionless relative phase velocity in the cross section of the slits are illustrated in Fig. 3. On the whole, these profiles confirm that which has been stated earlier relative to the nature of phase slippage.

Neglecting the phase flows in the thin layer at the wall, i.e., $0 < y < y_0$, it is thus not difficult to derive a formula for δ from (14). Using (20), for the case in which $Re_* \gg 1$, subsequent to integration we obtain

$$\delta \approx \varkappa U\,\mathrm{Re}_*\,(\ln \mathrm{Re}_*)^{-1},\quad \Pi/\sqrt{\mathrm{Re}_*} \gg 1$$

and, in particular, for the ratio of phase velocities averaged through the cross section, with consideration of the definition of U in (11), we have

$$\frac{\langle w \rangle}{\langle v \rangle} \approx 1 - \frac{2\varkappa}{9}\frac{\rho_1}{\rho_0}\left(\frac{a}{L}\right)^2\frac{\mathrm{Re}_*}{\ln \mathrm{Re}_*}. \tag{21}$$

Analogously,

$$\delta \approx 2\sqrt{\varkappa}\,U\,\mathrm{Re}_*^{3/2}/\Pi \ln \mathrm{Re}_*,\quad \Pi/\sqrt{\mathrm{Re}_*} \ll 1$$

and

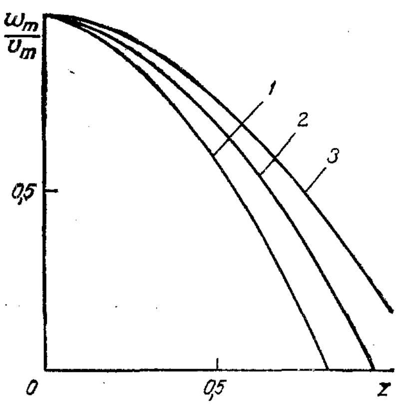

Fig. 4. Phase velocity ratio at the center of the slit as a function of the parameter Z: 1-3) $\log(L/a) = 3$, 4, 5.

$$\frac{\langle w \rangle}{\langle v \rangle} \approx 1 - \frac{12.8\sqrt{\varkappa}}{27\pi b} \frac{\rho_1}{\rho_0} \left(\frac{a}{L}\right)^3 \frac{\mathrm{Re}_*^{3/2}}{\ln \mathrm{Re}_*}. \tag{22}$$

The formulas corresponding to (21) and (22) for the ratio of particle and fluid velocities in the center of the slits can be obtained directly from (20). With arbitrary values for the ratio $\Pi/\sqrt{\mathrm{Re}_*}$ we have

$$\frac{w_m}{v_m} \approx 1 - U \frac{\mathrm{Re}_*}{\varkappa^{-1} \ln \mathrm{Re}_* + C}, \quad v_m = \frac{1}{\varkappa} \ln \mathrm{Re}_* + C. \tag{23}$$

Let us rewrite this formula in the form

$$\frac{w_m}{v_m} \approx 1 - \frac{K(\rho_1/\rho_0) Z^2}{\ln(L/a) + \ln Z_0 + \ln Z}, \tag{24}$$

$$K = \varkappa/9, \; Z_0 = \exp(\varkappa C/2), \; Z = (a/L)\sqrt{\mathrm{Re}_*},$$

using the definition of U and v_m and introducing the parameter Z, usually used for correlation of experimental data (see, for example, [6]). The theoretical relationships between w_m/v_m and Z are shown in Fig. 4. These are quite similar to the corresponding experimental curves from [6, 7, 9]. (Direct comparison of theory with experiment was not undertaken, since such curves are usually obtained for flows in a circular tube.) For gas suspensions the quantity from (24) becomes the negative quite rapidly as Z increases. To a considerable extent this represents something that is associated with our assumption of the total attraction of the particles by turbulent moles, which can hardly be valid in the case of rather large Re_*, particularly in the case of particles suspended in gases.

Let us dwell in some detail on the physical significance of the first equation in (15), which follows out of the second equation in (2), and under the above assumptions can be presented in the form

$$d(-c\langle v_y'^2 \rangle)/dy + f_y = 0, \quad c = \rho_1 \varphi, \tag{25}$$

where in the left-hand side it is the sum of the "thermodynamic" and regular forces acting on a particle per unit volume of mixture that actually plays a role. The first of these terms is expressed in standard fashion in (25) in terms of the derivative of the effective "pressure" of the dispersion phase, representing the corresponding density flux tensor component of the particle pulsation momentum associated with the transport of these particles by means of pulsation motion. Having divided (25) by β which, in this case, plays the role of effective mobility, we derive an equation which can be presented in the form

$$-D\, dc/dy + c(W_t + W_S) = 0,$$

$$D = \frac{\langle v_y'^2 \rangle}{\beta}, \quad W_t = -\frac{1}{\beta}\frac{d\langle v_y'^2 \rangle}{dy}, \quad W_S = \frac{\gamma}{\rho_1 \beta}\left|\frac{dv}{dy}\right|^{1/2}(v - w). \tag{26}$$

This equation reflects the fact that the total mass flow of the dispersion phase is equal to zero. In this case the quantity D functions as the coefficient of turbulent particle diffusion in the transverse direction, while W_t and W_S represent the velocities of the turbulent and ascending migrations of the particles, respectively (we again borrow the terminology from [7]). The rate of turbulent migration, first of all, was apparently introduced into the analysis in [18, 19]. Let us take note of the fact that the introduction of the diffusion flow and the coefficient of diffusion in terms of the thermodynamic force acting on the diffusing particles corresponds to the classic Einstein method. An analogous method was recently used in [20] to describe the laminar flows of suspensions and gas suspensions in the presence of Brownian or pseudoturbulent particle motion.

In conclusion let us note the possible expedient trends in the subsequent development of the theory. First of all, of importance is the selection of an appropriate approximating expression for the velocity of single-phase turbulent flow wherein consideration is given both to the unique properties of the viscous and transition layers, as well as to the absence of velocity in the core of the flow. No less important is information regarding the distribution of the Reynolds stresses in the flow, which, in addition to everything else, makes it possible to determine the tensor of the coefficients for the turbulent diffusion of particles.

The natural generalization of the theory must be clearly associated with consideration of the incompleteness of particle attraction by the turbulent moles and the reverse effect of the particles on the turbulence characteristics of the fluid, as well as with refinement of the expression for the transverse force acting on the particle. It is clear that the Saffman formula employed here is valid only in the case of a strictly laminar shear flow. Since it is nonlinear with respect to the velocity fields, the result of its averaging with respect to the turbulent flows does not, generally speaking, coincide with the force calculated directly for the average fields.

Finally, the proposed interpretation of particle behavior in turbulent flows leads to a new method of describing their turbulent diffusion, fundamentally distinguished from those with which we are familiar [7, 12]. Indeed, first of all, the diffusion phenomena are naturally included in the dynamic description of the motion of the dispersion phase by means of the Reynolds equations and, secondly, from the latter we derive the equations of the first order, but not of the second order, for those particle concentrations which are easily solved by the characteristic method.

NOTATION

A, constant in (7); a, particle radius; b, $\langle v_y'^2\rangle/u_*^2$; c, particle wake concentration; D, diffusion factor; F, f, functions introduced into (7); f_x, f_y, force components of the interphase interaction; K, coefficient in (24); L, half-widths of the slits; p, static pressure; q, relative volumetric flow of the dispersion phase; Q, U, parameters determined in (10) and (11); u_*, dynamic velocity; v, w, average fluid and particle velocities; W_t, W_S, migration velocities, introduced into (26); x, y, longitudinal and transverse coordinates; Z, Z_0, quantities introduced in (24); β, γ, coefficients in (3) and (4); δ, parameter determined in (14); $\eta = Re_* y/L$; κ, Karman constant; μ_0, ν_0, dynamic and kinematic fluid viscosity; $\xi = y/L$; ρ_0, ρ_1, fluid and particle-material densities; Π, parameter determined in (17); φ, particle volume concentration; $Re_* = u_* L/\nu_0$, dynamic Reynolds number; the angle brackets denote averaging over the cross sections; the subscript m pertains to velocities determined at the center of the slit.

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